MODELING OF CANTED COIL SPRINGS AND KNITTED SPRING TUBES AS HIGH TEMPERATURE SEAL PRELOAD DEVICES

by

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Abstract

by

Jay Joseph Oswald

Future reusable launch vehicles will require advanced structural seals. Seal protection will be needed for reentry vehicle control surfaces such as moveable flaps and elevons and hypersonic engine edge and hinge line seals. These seals must remain in sealing contact with opposing surfaces over multiple missions, even though the seal gap may be opening and closing due to thermal and structural loads. To meet this requirement seals must be resilient. Case Western Reserve University is working with NASA’s Glenn Research Center to develop more resilient, high temperature seals and preload devices.

This study details the creation of finite element models for two high temperature resilient structures: canted coil springs and knitted spring tubes. Each model was verified by comparison to experimental results of compression tests performed at Glenn Research Center. The models reveal the locations of stress concentration, and they show the force versus displacement relationship for each structure. These models provide a means for optimizing the design of canted coil springs and knitted spring tubes to advance NASA’s high temperature seal capabilities.
Chapter 1

Introduction

1.1 High Temperature Seals

The improvement of high temperature structural seals is critical to the development of future space vehicles. Advanced seal designs are required to function at higher operating temperatures and heat fluxes, conform to movement of opposing sealing surfaces, and operate without replacement or repair for up to ten times as many missions than current Shuttle orbiter seals are subject to. Current Shuttle orbiters require seals for the Shuttle elevons and body flaps. The depth of section is large enough in these locations that the seals can be recessed far enough away from the outer mold line to be subjected to relatively low temperatures ($>$1500 °F$^1$). Smaller reentry vehicles would have less available space, forcing the seals closer to the hot reentry gases passing over the vehicle. Additionally the Shuttle’s highly insulating tile system blocks heat from being conducted to the seals, while newer vehicles are embracing hot CMC structures. These conditions increase the operating temperature of the seals to greater than 2000 °F. Rudder/fin seals on the X-38 crew return vehicle were expected to reach 1900 to 2100 °F.
1.1.1 Seal Construction

Although there are many different seal configurations being evaluated there are common features for all designs. Each seal has a resilient component which provides a restoring force under compression. This component can either be an integral part of the seal, such as the Inconel X-750 knitted spring tube within a ceramic braided rope seal, or can be placed behind the seal as a seal preload device. Ceramic wafer seals, which by themselves are not compliant at all, can be installed in a groove behind a canted coil spring which provides a preload force on the coils and acts as the compliant component of the seal.

1.1.2 Seal Resiliency

A common failure mechanism of high temperature seals is the loss of resiliency with exposure to high temperature. This loss of resiliency is caused by the temperature dependent yield and creep strength of the material.

The current seal design used in several locations on the Space Shuttle orbiters, (including main landing doors, the orbiter external tank umbilical door, and the payload bar door vents), has a nominal diameter of about 0.62 in. and consists of an Inconel X-750 spring tube stuffed with Saffil batting and over-braided with two layers of Nextel 312 ceramic sheathing. Unfortunately this design loses its resiliency and suffers a large permanent set when compressed at high temperature (Figure 1.1).

Figure 1.1: Seal with baseline Inconel X-750 spring tube before and after compression testing at 1900 °F.
References


Chapter 2

Seal Preload Devices

The function of a seal preload device is to provide a contact force between a seal and the sealing surface. The greatest challenge is to provide this force at high temperature while the seal conforms to a moving surface.

2.1 Canted Coil Springs

Unlike typical compression springs that generate increasing amounts of force as they are compressed, the force produced by a canted coil spring is nearly constant over a large range of deflection as shown in Figure 2.1. The spring is loaded transversely and upon compression, the angle between the coils and the vertical axis increases. Canted coil springs can be produced in long segments that are ideal for installation in a groove as a seal preload device. This configuration would require very few canted coil springs to accomplish what otherwise could require hundreds of compression springs.
Figure 2.1: **Schematic of deflection response (bottom) of a typical canted coil spring (top).**

Currently canted coil springs are only available for relatively low temperature service (up to 575 °F). NASA’s design requirements, which are listed in table 2.1, require that advanced canted coil springs be designed. NASA has already considered refractory metals such as Mo-47.5Re (molybdenum rhenium) and Mo-0.5Ti-0.08Zr (TZM). These materials must be coated to prevent oxidation at high temperatures. Preliminary analyses suggest that canted coil springs can meet design requirements but will require advanced materials and optimized design.
<table>
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<th>Goal</th>
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<td>Operating Temperature</td>
<td>2000+ °F in oxidizing environment</td>
</tr>
<tr>
<td>Life</td>
<td>50+ heating cycles</td>
</tr>
<tr>
<td></td>
<td>100+ mechanical loading cycles</td>
</tr>
<tr>
<td>Deflection/stroke</td>
<td>20 percent of height</td>
</tr>
<tr>
<td>Permanent set</td>
<td>Less than 20 percent of stroke</td>
</tr>
<tr>
<td>Load range</td>
<td>2 to 10 lb per linear inch</td>
</tr>
<tr>
<td>Flexibility</td>
<td>Accommodate structural non-conformities</td>
</tr>
<tr>
<td></td>
<td>and seal around corners</td>
</tr>
</tbody>
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Table 2.1: Summary of canted coil spring design requirements.

2.2 Knitted Spring Tubes

An investigation into the performance of the baseline Shuttle rope seal by Dunlap et al.\textsuperscript{2} demonstrated that when the seal was compressed and heated up to 1900 °F, the seal suffered permanent deformation. Further investigations by DeMange et al.\textsuperscript{1} on the baseline Inconel X-750 knitted spring tube showed the spring tube itself suffered permanent deformation when compressed at temperatures as low as 1200 °F. The failure of the spring tubes to deform elastically at elevated temperatures was found to be caused by the temperature dependant strength of Inconel X-750. Taylor et al.\textsuperscript{5} showed that substituting the material Rene 41, a nickel-based high temperature, high strength alloy, increased the high temperature service temperature considerably (approximately 275 °F). Further improvements to the temperature capabilities of the knitted spring tube require both improved materials and optimized geometry.

References


Chapter 3

Canted Coil Spring Tube Analysis

3.1 Introduction

A finite element model was created to predict the performance of different canted coil spring designs and to estimate stress and contact force under loading. This model represents the properties of a single coil at the center of an infinitely long spring. Each canted coil spring design is defined by five geometry parameters. These parameters are used to create a finite element model of the spring deformed by prescribed displacements at the top and bottom surfaces of the spring.

3.2 Computational Model Description

The centerline of a single coil of a canted coil spring can be defined by the parametric equations:

\[
\begin{align*}
  x(t) &= (r_x - r_w) \sin(2\pi\omega t) \\
  y(t) &= (r_y - r_w) (1 - \cos(2\pi\omega t)) \\
  z(t) &= t + \frac{c}{2} (1 - \cos(2\pi\omega t)) \\
  t &\in (0, \frac{1}{\omega})
\end{align*}
\]

(3.1)

\(r_x\) - coil half width, (edge to edge)

\(r_y\) - coil half height, (edge to edge)
$r_w$ - spring wire radius

$\omega$ - number of coils per unit length

$c$ - cant amplitude (axial distance the top coil is shifted compared to a helical spring)

$t$ - the parameter variable.

As $t$ varies from zero to $\frac{1}{\omega}$, the inverse of the number of coils per unit length, the centerline of the spring is swept out through a single coil. The parameters are further defined in (Figures 3.1 and 3.2).

![Diagram of canted coil spring parameters](image)

Figure 3.1: Canted coil spring parameters (side view).
Some manufacturers define the front and back angles rather than coils per inch, \( \omega \), and cant amplitude \( c \). The following set of equations relates these two sets of parameters:

\[
\theta_{f,b} = \tan^{-1} \left( \frac{c \pm \frac{1}{2\omega}}{2(r_y - r_w)} \right) \tag{3.2}
\]

\( \theta_f \) - front angle  
\( \theta_b \) - back angle

Another parameter used in this analysis is the eccentricity of the spring. This parameter varies from zero to one, zero indicating a circular cross section, and one indicating a spring of zero height. As the eccentricity increases, the spring becomes more elliptical. The eccentricity of the spring is defined in the following equation, taken from the definition of eccentricity of an ellipse and assuming that the spring height is always the minor diameter.

\[
e \equiv \sqrt{1 - \frac{(r_y - r_w)^2}{(r_x - r_w)^2}} \tag{3.3}
\]
3.2.1 Boundary Conditions

Periodic symmetry is implemented by coupling nodes at the end faces of a coil. This reduces the number of elements required to accurately model stress distributions across the spring. Boundary conditions are imposed on the bottom and top cross sectional faces of the spring as shown in Figure 3.3. Displacements are prescribed as a ramp function applied to the center of the top face of the spring in the vertical direction. Since the top face of the spring is unconstrained in the axial direction, this model assumes negligible frictional forces. Stresses are calculated as percentages of the elastic modulus of the material at operating conditions, so the operating temperature does not influence the model behavior under the assumptions taken. At each load step, the reaction force at the displaced top node is recorded to estimate the spring stiffness as a function of displacement.

![Figure 3.3: Canted coil spring analysis boundary conditions.](image)

3.2.2 Finite Element Mesh

A mesh generator, (described in more detail in Appendix A), generates a mesh of 20-node elements from the parametric geometry of the canted coil spring. The density
of this mesh is controlled by the number of sections along the wire and the number of elements per section face. Mesh refinement is also discussed in Appendix A.

### 3.3 Comparison of Solutions to Experimental Results

A simplifying assumption used in the analyses is that the spring is long enough that, (1), the variation of load between coils is negligible, (2), end effects are negligible to global spring behavior, and (3), there are enough coils to provide adequate frictional load to prevent the spring from compacting axially while under load. Axial compression under transverse loading was observed when a canted coil spring has less than a critical number of coils. The experimental results presented in figure 3.4 are all with spring samples of 4.5 inches in length (approximately 40 coils of spring), which did exhibit axial compacting while being compressed. The testing was limited to samples that would fit between the loading platens. The experimental procedures and results of canted coil spring performance can be found in Dunlap et al. The spring samples tested were from heavy, medium, and light springs made of cold worked 302 stainless steel. The dimensions and parameters of each spring is presented in table 3.1.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Light</th>
<th>Medium</th>
<th>Heavy</th>
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<tr>
<td>Height (in.)</td>
<td>0.450</td>
<td>0.450</td>
<td>0.450</td>
</tr>
<tr>
<td>Length (in.)</td>
<td>0.518</td>
<td>0.508</td>
<td>0.510</td>
</tr>
<tr>
<td>Cant Amplitude (in.)</td>
<td>0.248</td>
<td>0.232</td>
<td>0.230</td>
</tr>
<tr>
<td>Wire Diameter (in.)</td>
<td>0.031</td>
<td>0.041</td>
<td>0.051</td>
</tr>
<tr>
<td>Coils per inch (in.(^{-1}))</td>
<td>14.47</td>
<td>9.213</td>
<td>8.297</td>
</tr>
<tr>
<td>Front Angle (°)</td>
<td>34</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>Back Angle (°)</td>
<td>27</td>
<td>23.5</td>
<td>23</td>
</tr>
<tr>
<td>Eccentricity (n/a)</td>
<td>0.510</td>
<td>0.483</td>
<td>0.494</td>
</tr>
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Table 3.1: Canted coil spring parameters for the light, medium, and heavy spring.
3.3.1 Importance of Spring Length and End Effects

Although a finite length spring was not considered analytically, the effect of spring length on behavior was investigated experimentally. Longer loading platens were constructed to allow springs of up to 18 inches of length to be tested. Each spring was compressed transversely with a procedure as described in Dunlap et al\textsuperscript{2}, except that each sample was subjected to only one loading cycle and no constraint was required to keep the spring from "walking" axially.

The original sample was composed of 214 coils. After each compression, approximately 10 coils were removed and the procedure repeated. Average force per coil vs. displacement results from samples having various number of coils are presented in figure 3.5.
The results show that shorter springs are less stiff and exhibit more nonlinearity. It was also observed that they compressed axially under loading. Light springs with more than 80 coils begin to act as they are infinitely long.

### 3.3.2 Modeling of Simple Friction Effects

A simple Coulomb friction model was applied to the top contact of the spring to better account for the frictional forces encountered as the spring is compressed. This model, (Figure 3.6), represents frictional force as the product of a coefficient of friction, \( \mu \), and the normal force acting at the surface, \( N \).

\[
F_{\text{friction}} = \mu N
\]  

(3.4)
Figure 3.6: Friction model applied on canted coil spring.

The results of an analysis of the light spring are compared to experiment in figure 3.7. The experimental data is taken from a compression test on a spring with 189 coils (approximately 13” in length). The bottom loading surface was a sandblasted aluminum which did not permit sliding during compression. The top loading surface was polished aluminum with a surface roughness in the direction of sliding less than 5µm. Comparing the experimental and computational results shows that the best approximation of the coefficient of friction is between 0.2 and 0.3.
Figure 3.7: Computational results with different coefficients of friction compared to experimental data

3.4 Canted Coil Spring Behavior

In this analysis, when the spring is deformed the top of the coils slide against the contact surface and the bottom coils rotate about their axis. This represents a configuration where the bottom of the spring is constrained axially or where the coefficient of friction is greater at the contact between the spring and bottom surface than between the spring and the top surface. The results would be identical if the bottom of the spring was allowed to slide and the top was constrained, so the motion of the spring is chosen solely as a convention.

As the canted coil spring is deformed, each coil rotates about the centerline of the wire at the bottom of the coil as shown in figure 3.8. Both sides of the coil can be approximated as semi-elliptic beams, (A and B), rotating about their bottom ends and joined at the top ends. Since the beams have different centers of rotation and lengths, the circular paths, (paths A and B), traced by their rotations are not coincident, and both beams deform such that their ends follow a common path (path C). The exact position of path C is dependent on the relative stiffness of beams A
Beam A must extend and beam B must be compressed as their ends follow the same path. Under this deformation the outer surface of B and the inner surface of A will be in tension, and the inner surface of B and the outer surface of A will be in compression as shown in figure 3.9. These predictions are verified by the finite element analysis stress distribution results on the inside and outside of the coil, presented in figure 3.10.
Figure 3.10: Finite element results showing $\sigma_{yy}$ stress on left and right sides of a canted coil spring.

3.5 Canted Coil Spring Parametric Study

To determine the effect of varying spring design parameters, a study was conducted where several parameters were varied and the response of the spring recorded. Table 3.2 lists the baseline design and the design parameters that are varied in the study as well as the limits of variation. In this study, each parameter was exercised through the limits listed in the table while all other parameters are held constant. The five independent parameters are wire diameter, cant amplitude, coils per inch, size (controlled by spring width), and eccentricity.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Baseline</th>
<th>Min</th>
<th>Max</th>
<th>Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire Diameter (in.)</td>
<td>0.041</td>
<td>0.026</td>
<td>0.056</td>
<td>0.005</td>
</tr>
<tr>
<td>Cant Amplitude (in.)</td>
<td>0.232</td>
<td>0.132</td>
<td>0.332</td>
<td>0.050</td>
</tr>
<tr>
<td>Coils per inch ($in.^{-1}$)</td>
<td>9.213</td>
<td>8.713</td>
<td>9.713</td>
<td>0.250</td>
</tr>
<tr>
<td>Size (width) (in.)</td>
<td>0.400</td>
<td>0.300</td>
<td>0.500</td>
<td>0.050</td>
</tr>
<tr>
<td>Eccentricity (in.)</td>
<td>0.526</td>
<td>0.264</td>
<td>0.782</td>
<td>0.052</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of design parameter variations from baseline design.

To compare the force and stress relationship for each parameter, results at 50 percent compression are reported for maximum Von Mises stress and force. The Von Mises stress is non-dimensionalized as a percentage of the elastic modulus of the material. These results are plotted in the following pages with the x-axis giving the parameter being examined, and separate y-axes for stress and force.
3.5.1 Flatness Parameter

To compare the shape of the force-displacement relationship a parameter is defined that quantifies the flatness of the curve. Flatness is desired because it allows for a more near-constant force over a wide range of displacement. This flatness parameter \((Fl)\) is defined by:

\[
Fl = \frac{1}{\Delta_f} \int_0^{\Delta_f} \frac{F(\Delta)}{F_{\text{max}}} d\Delta
\]  

\((3.5)\)

- \(Fl\) - the flatness parameter
- \(\Delta\) - displacement applied to the spring
- \(\Delta_f\) - the maximum displacement applied to the spring
- \(F(\Delta)\) - the contact force at a given displacement
- \(F_{\text{max}}\) - the maximum contact force

This parameter is equivalent to the ratio of the strain energy of the spring and the strain energy of a spring with a step function force-displacement curve. Since the ideal force-displacement curve is a step function, any spring that could achieve this behavior would have a flatness of 1 and would create a constant force over the entire displacement range. If the spring acted as a helical spring and had a linear force displacement relationship, the flatness would be 0.5.

3.5.2 Wire Diameter

The stiffness of a canted coil spring is proportional to the fourth power of the wire radius and the stress in the wire is linearly proportional to the wire radius (Figure 3.11). The higher order relationship to stiffness is due to the second moment of area about the neutral axis of a round beam, which is proportional to the fourth power of its radius.
Figure 3.11: Effect of variation of spring wire diameter.

The peak stress on a round beam is defined by the relationship:

\[
\sigma_{\text{max}} = \frac{Mr}{I}
\]  
(3.6)

\(M\) - Moment on the beam (the product of force and length)

\(r\) - Wire radius (maximum distance to the neutral axis)

\(I\) - Area moment of inertia about a circle \(\left(\frac{\pi r^4}{4}\right)\)

Comparing the results of the analysis, (Figure 3.11), with the simple beam bending model, (Equation 3.6), shows that the results are compatible when each term is replaced with its proportionality to the wire radius (Equation 3.7).

\[
\sigma = \frac{(FL)r}{I} \rightarrow (r) = \frac{(r^4)}{r^4}
\]  
(3.7)

Comparing the experimental results to a beam bending model is adequate to show the relationship of the radius to stress and force. For completeness another equation should show one of the two dependencies independently.
The deflection of a cantilever beam given an applied load is given as:

\[ d = -\frac{PL^3}{3EI} \]  \hspace{1cm} (3.8)

d - deflection of a beam (externally imposed, thus not dependent of the radius)

\( P \) - the force or load on beam

\( L \) - the length of the beam

\( E \) - the elastic modulus

\( I \) - The area moment of inertia about a circle (proportional to the fourth power of the radius)

Since the left hand side of equation 3.8 is independent of the radius, the right hand side must also be independent of the radius. The only two terms in the right hand side that have a radius dependence are the force and area moment of inertia. For the ratio of these to be not dependent on the radius, the force must be proportional to the fourth power of the wire radius.

This analysis requires the shape of the deformed spring to be independent of the radius of the wire. This requirement is satisfied as long as a beam model approximation is valid because a change in the radius will result in a uniform change in stiffness across the entire model. Varying the wire radius produces only a change in stiffness which is analogous to a change in the elastic modulus of the material. Since the wire diameter affects only the stiffness linearly, there is no change to the shape of the force-displacement relationship and the flatness is unchanged.

### 3.5.3 Cant Amplitude

Figure 3.12 shows that as cant amplitude is increased, the stiffness of the spring decreases, as well as the maximum Von Mises stress. Increasing the cant amplitude causes the front and back angles of the spring to increase. This causes a larger moment load about the bottom of each coil when the spring is loaded.
The cant amplitude has the greatest influence of all the design parameters on the flatness of the force displacement curve. Canted coil springs that have a very small cant amplitude exhibit much more nonlinearity and a larger displacement range of near constant force. Figure 3.13 shows the flatness parameter decreases with higher cant amplitude. As the cant amplitude approaches zero, the force-displacement relationship ceases to be monotonic and the spring becomes unstable past a critical deflection (Figure 3.14).
Figure 3.13: **Effect of cant amplitude on flatness of force-displacement curve.**

Figure 3.14: **Force-displacement relationship of a spring with a low cant amplitude.**
3.5.4 Coils Per Inch

When the number of coils per inch decreases from the baseline geometry, the stiffness of each coil increases (Figure 3.15). As the coils move farther apart, the ratio between the cant amplitude and the distance between coils decreases, which changes the behavior of the deformation from coil rotation to the crushing the coils. For the geometry considered in table 3.2, the force per inch of spring also increases as the number of coils decreases. This indicates that even as there are fewer coils per inch to carry load, each coil becomes much stiffer, and the overall stiffness per unit length of spring increases.

The flatness of the force-displacement relationship is not influenced by the number of coils per inch. The flatness parameter varies by 0.03 percent over the range of coils per inch considered.

![Figure 3.15: Effect of variation of coils per inch.](image)

3.5.5 Coil Size

The coil size was controlled by increasing both the spring width and height at constant wire thickness so that there was no change in eccentricity. The coil size has a nearly
linear influence on the maximum Von Mises stress and a higher order effect on the spring force (Figure 3.16). Both values decrease as the spring becomes wider. If the approximate model of two semi-elliptic beams is used, then increasing the size of the spring is analogous to increasing the offset of loading on an eccentrically loaded column, which acts to soften the structure.

![Figure 3.16: Effect of variation of spring coil size.](image)

### 3.5.6 Spring Eccentricity

To model different spring eccentricities, the height of the spring is adjusted while holding all other parameters constant. Since height is the minor diameter, an increase of height leads to a decrease in spring eccentricity. Figure 3.17 shows the influence of changes in eccentricity from the baseline spring model on stress and force at 50 percent compression. As the spring becomes more elliptical, (i.e., eccentricity increases), maximum stresses decrease sharply while the stiffness of the spring increases. The drawback of this manipulation is that as the eccentricity increases the length of the available spring stroke decreases with the height of the spring.
The flatness of the force-displacement relationship is barely influenced by the eccentricity. Figure 3.18 shows that the flatness parameter decreases slightly as the eccentricity is increased. The flatness parameter varies by about 5 percent over the range of eccentricity considered.
Figure 3.18: Effect of spring eccentricity on flatness of force-displacement curve.

3.6 Optimization Strategy for Canted Coil Springs

The optimization strategy for a canted coil has been developed to maximize spring stiffness and force-displacement flatness for a given allowable stress. The procedure is developed by maximizing a variable to quantify the combination of flatness and stiffness into a single scalar for each spring tube design. This metric is defined by the follow expression:

$$ s = (F) (Fl)^w $$  \hspace{1cm} (3.9)

*s* - score variable that is to be maximized via optimization

*F* - force at a designated compression

*Fl* - flatness factor

*w* - flatness weighting factor
The flatness weighting factor changes the importance of flatness with respect to the force. If this number is zero, then the flatness is ignored. A value less than unity favors the maximization of force over flatness, while a value greater than one places greater emphasis on increasing the flatness of the force-displacement relationship.

The performance metric is calculated from each analysis and can be represented as a function of the design variables. Since the wire diameter is separable from the stiffness and does not impact the flatness of the force-displacement relationship, it is separated from the analysis.

The optimization routine starts with a baseline design. The values of the design variables are not important as long as they are all within the specified design constraints. From this initial design, a set of designs are created with small perturbations from the baseline parameters. This results from this set will be used to approximate the first and second derivatives of the score variable with respect to the design variables.

The entire set of designs are then created as finite element models, and each is given the same displacement loads. A post processing script then returns the maximum Von Mises stress and resulting contact force at incremental displacement steps.

The relationship between wire radius, force, and stress is then used to convert the results to a constant final maximum stress. The force at each iteration is scaled by:

\[
F_{i,new} = \left( \frac{\sigma_{v,ref}}{\sigma_{v,max}} \right)^4 F_i
\]  

\(F_{i,new}\) - the scaled force at each load step for a reference stress.

\(F_i\) - the contact force at each load step

\(\sigma_{v,ref}\) - the reference stress (normally the highest allowable design stress)

\(\sigma_{v,max}\) - the maximum stress at the highest load step
If the maximum $\sigma_v$ reaches a maximum before the final load step, then the highest maximum $\sigma_v$ should be used for $\sigma_{v_{\text{max}}}$ during loading. This ensures that the spring will never exceed $\sigma_{v_{\text{ref}}}$ during loading.

The initial design wire diameter is then replaced with the wire diameter needed to scale the results (Equation 3.11). Replacing the wire diameter at each iteration helps to keep any error associated with separating the wire diameter from the analysis small since its value is updated at each step.

\[
    r_{w_{\text{new}}} = \left( \frac{\sigma_{v_{\text{ref}}}}{\sigma_{v_{\text{max}}}} \right) r_w
\]

Once the forces are scaled for the reference stress, the flatness factor (Equation 3.5) and score (Equation 3.9) are calculated for each of the designs in the set. Since the response to variation of the parameters is smooth and relatively well behaved, a simple Newtonian method for finding extrema will converge to an optimal value with relatively few iterations.

The score of the each subsequent design is then compared to the score of the previous iteration to determine whether or not the design process has reached convergence. If the difference between scores is negligible, then optimization process stops and the design is output as the final iteration. If the difference is not negligible, the baseline design is replaced with the current iteration and the process is repeated. This entire process is illustrated in figure 3.19.
3.6.1 Summary and Conclusions of Canted Coil Spring Analysis

The finite element model approximation of a long spring gives a good approximation to the shape and magnitude of the force vs. displacement response measure for
a canted coil spring segment under transverse compression loading. This model is further improved by considering the effect of the length of the actual spring segment and by including frictional effects in the analysis. The model is useful for predicting the influence of changing any of several parameters, as well as estimating the values of stress in the spring, which can not be easily measured. Additionally the model shows that several parameters have different order relationships to force and maximum Von Mises stress. With an optimization strategy, designs can be generated to maximize performance without exceeding material limitations. Based on these results, the following conclusions are noted:

1. The behavior of a canted coil spring under compression depends on its length. While a theoretical model of an infinitely long spring gives accurate predictions of force and stress in long canted coil spring segment, it does not account for the effects of unconstrained ends.

2. Unlike helical compression springs that store energy in shear strains (torsion), the energy storage mechanism in canted coil springs is primarily one of bending of the semi-elliptic segments. The loading behavior is similar to that of an eccentrically loaded column, where after a critical force, displacement is unconstrained. Coil binding, or contact between adjacent coils bounds the displacement for a force above the critical load level.

3. The stiffness of the spring is highly dependant on the wire diameter and the cant amplitude. Another factor in the spring stiffness is the amount of deformation required in the semi-elliptic beam segments to allow for the rotation of each coil. This deformation can be approximated by the amount that the arcs of the semi-elliptic segments would diverge from each other if they were not constrained to follow the same path. Combining the load versus displacement relationship of a curved buckling column with the kinematics of the spring deformation may yield an analytic solution to the behavior of the spring.
4. The behavior cannot be fully described by a model with independent parameters. Any optimization strategy should take into account the effects of combined variation of parameters in a multi-factor analysis.

References


Chapter 4

Knitted Spring Tube Analysis

4.1 Introduction

A finite element model of a knitted spring tube was created to evaluate stress and contact force during compression of the spring tube. The objectives of the model are to guide spring tube design by optimizing the geometry, and to provide stress predictions to aid in material selection. The model also helps to develop an understanding of the deformation mechanisms taking place as the spring tube is compressed, and yields insight into how the variation of several parameters affects performance.

4.2 Computational Model Description

4.2.1 Spring Tube Knit Geometry

A knitted spring tube, (Figure 4.1), consists of a series of needles interwoven about a base helix. To simplify the description of the spring tube geometry, the needle pattern is defined separate from the base helix.
The needle pattern is defined by the combination of a circular section and a linear section. These two functions are piecewise continuous and smooth at their intersection. (Equation 4.1) gives a definition of the needle geometry (Figure 4.2). The quarter needle pattern is then repeated (with mirroring and translation) to create a knit pattern (Figure 4.3).
Figure 4.2: Parametrization of a quarter needle

\[ \tilde{P}_{\text{knit}} = \begin{cases} \tilde{f}_1(t) & = \begin{bmatrix} at \\ bt \end{bmatrix} \begin{bmatrix} \tilde{\epsilon}_1 & \tilde{\epsilon}_2 \end{bmatrix} t \in [0, T_A] \\ \tilde{f}_2(t) & = \begin{bmatrix} x_c + r \sin(\omega (t - T_A) - \alpha) \\ y_c - r \cos(\omega (t - T_A) - \alpha) \end{bmatrix} \begin{bmatrix} \tilde{\epsilon}_1 & \tilde{\epsilon}_2 \end{bmatrix} t \in [T_A, T_f] \end{cases} \] (4.1)

\( \tilde{f}_1 \) - linear part of the needle

\( \tilde{f}_2 \) - circular part of the needle

\( t \) - the parameter variable

\( \tilde{\epsilon}_1 \) - direction vector parallel to spring tube axis

\( \tilde{\epsilon}_2 \) - direction vector tangent to spring tube and perpendicular to spring tube axis
\(a\) - linear constant for \(\tilde{f}_1\) in the \(\tilde{e}_1\) direction

\(b\) - linear constant for \(\tilde{f}_1\) in the \(\tilde{e}_2\) direction

\(T_A\) - value of the parameter variable at the transition from \(\tilde{f}_1\) to \(\tilde{f}_2\)

\(T_f\) - value of the parameter variable at the end of \(\tilde{f}_2\)

\(x_c\) - position in the \(\tilde{e}_1\) direction of the center of the needle circular segment

\(y_c\) - position in the \(\tilde{e}_2\) direction of the center of the needle circular segment

\(r\) - the needle radius

\(\omega\) - variable which defines the rate of curvature of the needle radius with respect to the parameter variable

\(\alpha\) - angle that the circular segment extends past a quarter circle.
Figure 4.3: **Parametrization of knit pattern**

The location of the center of the circle segment is given by the variables $x_c$ and $y_c$, which are calculated by the following expressions:

\[
x_c = \frac{1}{2} \left( \frac{1}{\text{cpi}} - \delta \right) \tag{4.2}
\]

\[
y_c = \frac{\pi D}{4 \text{npt}} \tag{4.3}
\]

- $\text{cpi}$ - number of courses (turns) per linear inch of spring tube
- $\delta$ - distance between adjacent needle centers
- $D$ - diameter of the spring tube
- $\text{npt}$ - number of needles per turn (course)
The final variable that is specified is the parameter $T_f$ which is arbitrarily defined as the increment of variable $t$ to trace out a quarter of a needle if one inch of knitted spring tube is traced out over the range (0-1). This relation is expressed by:

$$T_f = \frac{1}{4} \left( \frac{1}{npt} \frac{1}{cpi} \right) \quad (4.4)$$

Since five variables remain that are not yet defined either implicitly or explicitly five equations are needed for enforcing continuity and smoothness and the constraint that the point where $t$ is equal to $T_f$ be at the top of the circle segment. The five required constraints are given below: Equations 4.5 and 4.6 satisfy continuity.

$$a T_A = x_c - r \sin (\alpha) \quad (4.5)$$
$$b T_A = y_c - r \cos (\alpha) \quad (4.6)$$

Equations 4.7 and 4.8 satisfy smoothness.

$$a = \omega r \cos (\alpha) \quad (4.7)$$
$$b = -\omega r \sin (\alpha) \quad (4.8)$$

If the magnitude of the derivatives is equal for all values of $t$ in either function, then the distance along the curve in a parameter increment $\Delta t$ will be equal at all points along the curve. Since both functions have a constant magnitude derivative at all values of $t$, (Equation 4.7) and (Equation 4.8) force a constant length segment for a $\Delta t$ at any point along the curve. The final constraint equation is required to force the point at $T_f$ to be at the top of the circle.

$$\omega (T_f - T_A) - \alpha = \frac{\pi}{2} \quad (4.9)$$

There are now 5 constraint equations (Equation 4.5 - Equation 4.9), and 5 unknowns, \{a, b, \omega, T_A, \alpha\}, in the parametric definition of the spring tube knit geometry. The method of solving for each of the unknowns is as follows:
Rearranging (Equation 4.5) and (Equation 4.7):

\[ \omega r \cos (\alpha) T_A = x_c - r \sin (\alpha) \rightarrow T_A = \frac{x_c}{\omega r \cos (\alpha)} - \frac{\tan (\alpha)}{\omega} \quad (4.10) \]

Rearranging (Equation 4.6) and (Equation 4.8):

\[ -\omega r \sin (\alpha) T_A = y_c - r \cos (\alpha) \rightarrow T_A = \frac{-y_c}{\omega r \sin (\alpha)} + \frac{1}{\omega \tan (\alpha)} \quad (4.11) \]

Setting the right hand sides of (Equation 4.10) and (Equation 4.11) equal to each other and rearranging allows for a numerical solution for \( \alpha \):

\[ x_c \sin (\alpha) + y_c \cos (\alpha) = r \quad (4.12) \]

Knowing \( \alpha \), (Equation 4.9) is solved for \( T_A \), and substituted into (Equation 4.5). The resulting equation is then solved for \( a \).

\[ a \left( T_f - \frac{\pi}{2} + \frac{\alpha}{\omega} \right) = x_c - r \sin (\alpha) \rightarrow a = \frac{x_c - r \sin (\alpha)}{T_f - \left( \frac{\pi}{2\omega} + \frac{\alpha}{\omega} \right)} \quad (4.13) \]

(Equation 4.13) is then combined with (Equation 4.7) which allows for a solution of \( \omega \).

\[ \frac{x_c - r \sin (\alpha)}{T_f - \left( \frac{\pi}{2\omega} + \frac{\alpha}{\omega} \right)} = \omega r \cos (\alpha) \rightarrow \omega = \frac{x_c - r \sin (\alpha) + r \cos (\alpha) \left( \frac{\pi}{2} + \alpha \right)}{r \cos (\alpha) T_f} \quad (4.14) \]

With \( \omega \) known the remaining unknowns can be calculated by simple substitutions.

### 4.2.2 Base Helix Geometry

The base helix around which the spring tube is wrapped is defined by the following equation:

\[
\bar{P}_{\text{Helix}} (t) = \begin{bmatrix}
D \sin (2\pi (cpi) t) \\
D \cos (2\pi (cpi) t) \\
(t)
\end{bmatrix}
\begin{bmatrix}
\tilde{x} \\
\tilde{y} \\
\tilde{z}
\end{bmatrix} \quad (4.15)
\]
At every point on the base helix, $\vec{P}_{Helix}(t)$, a local coordinate system is defined by three unit vectors. The knit geometry is then added to the base helix geometry with the vector transformations shown in equation 4.16. There is some distortion incurred from mapping a flat Cartesian surface onto a curved cylindrical surface, but since the magnitude of the knit geometry tangential vector is relatively small, compared to the radius of curvature of the helix, this distortion is negligible. Also due to the process by which the spring tube is manufactured, the distortion mimics the natural flattening of the needles. A drawing of the centerline of the wire, (Figure 4.4), shows the needles don’t fully conform to the spring tube curvature; instead they appear to be tangent. This effect is also visible in a photo of a spring tube (Figure 4.5), as each needle appears to flatten, making the end view of a spring tube appear as a series of linear segments.

Figure 4.4: Model geometry showing distortion at needles.
Figure 4.5: Photo showing distortion at needles.

\[\begin{align*}
\vec{e}_1 &= \hat{z} \\
\vec{e}_2 &= -\sin \left(2\pi (c\pi) (t)\right) \hat{x} + \cos \left(2\pi (c\pi) (t)\right) \hat{y} \\
\vec{e}_3 &= \vec{e}_2 \times \vec{e}_1
\end{align*}\] (4.16)

The first quarter needle of the knitted spring tube is calculated by:

\[
\vec{P}(t) = \vec{P}_{\text{Helix}} + \left( \vec{P}_{\text{knit}} - t \frac{\pi D}{4T_f n pt} \vec{e}_2 \right) \quad (4.17)
\]

The last term in (Equation 4.17), subtracting a vector in the second unit vector direction, accounts for the distance traveled along the helix in the tangential direction. Points beyond the first quarter needle are calculated using various symmetries, (Figure 4.6), present in the knitted pattern.
Figure 4.6: **Full needle created by two mirroring operations of a quarter needle.**

Although the needle interaction models considered do not include contact analysis, the centerline geometry was modified so that there is no initial contact or penetration of needle surfaces. To achieve a non-contact, non-penetrating geometry, the needles were given a radial displacement (Figure 4.7).

The radial displacement was calculated as a fourth order polynomial with three parameters, (Figure 4.8). The independent variable is the normalized axial distance along the needle. The first parameter, $\psi$, is the normalized distance along the coil where there is intersection. The second parameter, $\alpha$, is the radial distance outward the needle is displaced at the intersection. The third parameter, $\lambda$, is an inward radial displacement expressed in terms of the wire radius. The function has a value and first derivative of zero at the start of each needle, and has a maximum at $\psi$ at the tip of the needle. The start of the needle is the point at which the parameter $t$ is equal to zero.
4.2.3 Finite Element Mesh

The geometry was generated with the same meshing algorithm as used for the canted coil spring analysis. The elements making up the spring tube are all 20 node solid elements (SOLID95). These are chosen because they allow for accurate representation of curved boundaries with few elements, which is ideal for meshing a wire. They also give higher order approximations of displacement and stress than 8 node elements.
4.2.4 Node Coupling and Periodic Symmetry

The spring tube is modeled as a single course (one period of the base helix) to reduce the amount of time required for each analysis. Since all courses of the spring tube are identical except they are located at different relative axial positions, periodic symmetry can be applied by coupling nodes at the cut faces. Figure 4.9 shows a course of a spring tube cut such that there are no needles in contact at the cut boundary. Each cut face is then coupled with a face with the same normal vector. Lines connecting each cut face would all be parallel to the axial direction and each would be the same length (Figure 4.10). Figure 4.11 shows the periodic symmetry expanded.

Figure 4.9: Single spring tube course cut for periodic symmetry.
Figure 4.10: Single spring tube course shown with coupled face pairs drawn.
Figure 4.11: Full spring tube showing assembly of single course pattern.

4.2.5 Needle Interaction Models

To simulate the contact between adjacent needles two models were used. The first assumes negligible contact between needles, and the second assumes needles are bound together by two linked elements. The link elements are two node uniaxial force members. Their stiffness is greater than the axial stiffness of the spring tube wire by
a factor of 1000. This value was chosen assuming that contact stiffness is several orders of magnitude greater than the bending stiffness of the wire, and assumes the needles are completely bound to each other by contact and friction during the compression. The effect is a type of hinged joint between the two needles.

In contrast, the light contact model allows each needle to move freely and independently, (except for ends which are coupled due to symmetry). While both models avoid the need for a detailed analysis of friction, they each assume opposite extremes, (i.e. \( \mu = 0 \) for the free model and \( \mu = \infty \) for the linked model.) Figure 4.12 shows a schematic of the two needle interaction models.

![Figure 4.12: Needle interaction models showing (left) free behavior and (right) hinged behavior.](image)

### 4.2.6 Boundary Conditions

In operation, spring tube reinforced seals are compressed in a groove between two flat surfaces. This compression was simplified in the model with displacement boundary conditions simulating compression between two flat plates. Four nodes at the highest and lowest positions (Figures 4.13 and 4.14), were constrained. The lowest point (or one of the lowest points if there are many) was fixed in all directions to prevent rigid body motion. The remaining bottom points were fixed in only the y-direction.

The highest four nodes were given a prescribed displacement in the y-direction.
to simulate compression of the spring tube. This displacement is broken down into several load steps to speed convergence of the analysis as well as provide intermediate values of force and stress as the spring tube is compressed. One of the highest four nodes is also fixed in the x-direction. This constraint should not be required for convergence, but its inclusion decreases the amount of iterations required for convergence. It is justified assuming that the spring tube will not rotate about its axis as it is compressed. Figures 4.13 and 4.14 show the constrained nodes.

**Figure 4.13:** Spring tube top boundary conditions.

**Figure 4.14:** Spring tube bottom boundary conditions.
4.3 Comparison of Computational Results to Experimental Data

4.3.1 Knitted Spring Tube Test Specimens

Computational results were compared with experimental data as a means of verifying the computational model. Spring tube samples of Inconel-X750 with different knit geometries were tested in a compression fixture at NASA Glenn Research Center\(^3\). The specimens were cut to a nominal 4 in. length and had an outer diameter of 0.560 ± 0.025 in.

4.3.2 Test Procedures and Equipment

Each specimen was compressed between flat platens by a servo-hydraulic load frame. Displacement and contact force were recorded as the spring tube was compressed. A laser extensometer was used to measure displacement to an accuracy of ± 0.00025 in. Force was measured to within ± 0.04 lb with a 500 lb load cell calibrated to a ± 100 lb range. A more detailed description of this test rig can be found in Dunlap, et al\(^2\).

4.3.3 Comparison of Results

Three knitted spring tube specimens were compressed to determine their force-displacement relationships. To validate the computational spring tube models these data were compared with analytical results. The specimens tested were the baseline three wire spring tube design, and two modified designs made from a single wire. For each design, there are six independent variables that define the spring tube geometry. Of these six, four were specified directly, and the other two were implicitly determined. The specified variables were the spring tube diameter, \((D_{tube})\), the number of courses per inch, \((cpi)\), the number of needles per turn, \((npt)\), and the wire diameter, \((D_{wire})\). The implicit variables were determined from measurements of the spring tube geometry. The needle loop radius, \((r_{needle})\), is determined by the manufacturing process, and
cannot be specified. In the samples received there was a high degree of variance in the needle loop radius within each specimen. The values used in the analysis were the average of the actual values for each design. The distance between adjacent needle centers, \((\delta)\), determines how tightly needles are interwoven. This value was determined by manually adjusting the model from a measured approximation until the needle intersections matched the appearance of the actual spring tube.

The spring designs are denoted by their material and loop density except in the case of the three strand baseline design, which was named Inconel ST-5. The loop density was defined as the number of loops per square inch of spring tube by Taylor et al\(^3\) (Equation 4.18). This number was rounded to the nearest whole number.

\[
\text{Loop Density} = \frac{\text{cpi} \times \text{npt}}{\pi \text{D}_{\text{tube}}}
\]  

(4.18)

### 4.3.4 Single Wire Spring Tubes

The first two specimens were knitted from a single wire. Table 4.1 gives the parameters used to model each spring tube design.

<table>
<thead>
<tr>
<th>Specimen Identifier</th>
<th>(D_{\text{tube}})</th>
<th>cpi</th>
<th>npt</th>
<th>(D_{\text{wire}})</th>
<th>(r_{\text{needle}})</th>
<th>(\delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inconel 34 LD</td>
<td>0.560</td>
<td>6</td>
<td>10</td>
<td>0.009</td>
<td>0.059</td>
<td>0.053</td>
</tr>
<tr>
<td>Inconel 64 LD</td>
<td>0.560</td>
<td>7</td>
<td>16</td>
<td>0.009</td>
<td>0.043</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Table 4.1: Single Strand Spring Tube Specimen Parameters

These two spring designs are both considered relatively coarse in that there is a lot of empty space on the tube surface. In this case, it can be assumed that the needle interaction and contact between wires is relatively small and the free needle model will accurately represent the behavior of the spring tube. This assumption was verified by comparing experimental displacement versus contact force data to the analytical predictions of each model (Figure 4.15 and 4.16).
Figure 4.15: Results of Inconel 34 LD spring tube models compared with experimental data.

Figure 4.16: Results of Inconel 64 LD spring tube models compared with experimental data.
4.3.5 Three Wire Spring Tubes

The final specimen was knitted with three wires running parallel with each other along the knit pattern. Table 4.2 gives the baseline spring tube design parameters.

<table>
<thead>
<tr>
<th>Specimen Identifier</th>
<th>$D_{tube}$</th>
<th>cpi</th>
<th>npt</th>
<th>$D_{wire}$</th>
<th>$r_{needle}$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inconel ST-5</td>
<td>0.560</td>
<td>4.9</td>
<td>10</td>
<td>0.009</td>
<td>0.061</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Table 4.2: Triple Strand Spring Tube Specimen Parameters

Since there are three wires running along this pattern there is less empty space on the tube surface. This leads to more contact between wires and more interaction between needles. The results, (Figure 4.17, suggest that at some point during compression the behavior of the spring tube changes from small contact forces and negligible needle interaction to significant contact force and strong needle interaction. This transition is reflected by a change of slope in the experimental data. The lines superimposed on the experimental data illustrate this increase of stiffness due to contact. Table 4.3 gives the slopes of the experimental and analytical data.

![Graph](image)

Figure 4.17: Results of Inconel ST-5 spring tube models compared with experimental data.
<table>
<thead>
<tr>
<th>Line</th>
<th>Type</th>
<th>Slope (lbf/in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linked Model</td>
<td>Analytical</td>
<td>0.007</td>
</tr>
<tr>
<td>High Compression</td>
<td>Experimental</td>
<td>0.007</td>
</tr>
<tr>
<td>Unlinked Model</td>
<td>Analytical</td>
<td>0.004</td>
</tr>
<tr>
<td>Low Compression</td>
<td>Experimental</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 4.3: Triple Strand Spring Tube Specimen Parameters

4.4 Stress Contour Plots

4.4.1 Von Mises Stresses

The Von Mises stress is a stress-invariant which is useful for predicting failure of ductile materials which in the case of a spring tube are superalloy metals. The results shown are from the Inconel 64 LD spring tube. The locations in the spring of highest stress are identical for each spring tube modeled, so the 64 LD case is presented as a representative example. Figure 4.18 shows the maximum stress locations as the top and bottom of the spring tube. These results are consistent with the experimental observations of permanent deformation at the top and bottom of the spring tube (Figure 4.19).
Figure 4.18: Von Mises stresses of spring tube (Top view).

Figure 4.19: Deformed spring tube compared to original circular geometry.

Figure 4.20 gives a closeup view of peak stress locations near the top contact point of the spring tube. The stress distribution at the cut face indicates bending as the primary deformation mechanism, but also shows evidence of torsion caused by a moment parallel to the wire axis.
Figure 4.20: Von Mises stresses near top contact of spring tube.

Figure 4.21: Von Mises stresses of spring tube (Front view).

4.4.2 Principal Stresses

The principal stresses are useful for predicting failure of brittle materials, such as ceramics. As the required operating temperature of advanced seal designs increases, ceramics become more desirable for their high temperature strength. Figures 4.22-
4.27 show the first and third principal stresses for top, closeup top contact, and front views of the spring tube. The first principal stress is the maximum tensile stress, while the third principal stress is the maximum compressive value on the spring tube. Half of the difference between these two values gives the maximum shear stress on the spring tube.

Figure 4.22: First principal stresses of spring tube (Top view).
Figure 4.23: Third principal stresses of spring tube (Top view).

Figure 4.24: First principal stresses near top contact of spring tube.
Figure 4.25: Third principal stresses near top contact of spring tube.

Figure 4.26: First principal stresses of spring tube (Front view).
4.5 Effect of Variation of Design Variables on Stress and Stiffness

4.5.1 Variation of Wire Diameter

The stiffness of a knitted spring tube is proportional to the fourth power of the wire radius and the stress in the wire is linearly proportional to the wire radius (Fig. 4.28). The higher order relationship to stiffness is due to the second moment of area about the neutral axis of a round beam, which is proportional to the fourth power of the radius. This relationship is identical to that of the canted coil spring, or any small strain bending of a wire-based structure.

4.5.2 Variation of Courses per Inch and Needles per Turn

Current manufacturing capabilities limit control over the spring tube geometry to four parameters: the number of wires knitted in parallel, the number of courses per
inch (cpi), the number of needles per turn (npt), and the wire diameter. The wire diameter can be separated from the analysis since its relationship to stress and stiffness is known and independent of the other parameters. Since the numerical analysis does not address the number of wires knitted in parallel the stiffness is assumed to be proportional to the number of wires used and the parametric analysis is limited to cpi and npt. Furthermore the manufacturer of the spring tubes currently has the tooling capabilities to produce only spring tubes of either 10 or 16 npt for spring tubes with a nominal 0.560 inch diameter. An intermediate npt of 13 was modeled to determine whether npt had a significant impact on performance. Five different values of cpi were selected (4, 4.9, 6, 7, and 8). These produced 15 combinations of cpi and npt. For each combination, needle radius ($r_{\text{needle}}$), and needle spacing ($\delta$), were estimated.

The spring tube designs were ranked by the maximum Von Mises stress at 20 percent compression. To correct for the differences of contact force between the designs, the stress was scaled by a force correction factor. The force scaling factor, $f_{\text{Force}}$, was calculated as the ratio between the contact forces between each design and the baseline design raised to the one fourth power (Equation 4.19). This is equivalent

Figure 4.28: Wire Diameter vs. stress and contact force. The dashed lines show least squares best fits to the functions indicated.
to changing the wire diameter so that the force at 20 percent compression is the same for each design.

\[ f_{\text{Force}} = \left( \frac{F_{\text{Contact},i}}{F_{\text{Contact},0}} \right)^{\frac{1}{4}} \]  \hspace{1cm} (4.19)

Table 4.4 shows force and stress results from the parameter study. These results assume a single wire spring tube made from Inconel X-750. The baseline knit parameters are shown bold. The effective wire diameter, \( d_{\text{wire},\text{eff}} \), (to achieve constant force results) is also presented.

<table>
<thead>
<tr>
<th>cpi (in.(^{-1}))</th>
<th>npt</th>
<th>( r_{\text{needle}} ) in.</th>
<th>Force lbf/in</th>
<th>( \sigma ) ksi</th>
<th>( f_{\text{Force}} \sigma ) ksi</th>
<th>( d_{\text{wire},\text{eff}} ) in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10</td>
<td>0.069</td>
<td>0.096</td>
<td>36.8</td>
<td>41.8</td>
<td>0.0102</td>
</tr>
<tr>
<td>4.9</td>
<td>10</td>
<td><strong>0.060</strong></td>
<td>0.162</td>
<td>48.7</td>
<td>48.7</td>
<td><strong>0.0090</strong></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>0.056</td>
<td>0.251</td>
<td>59.6</td>
<td>53.4</td>
<td>0.0081</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>0.054</td>
<td>0.348</td>
<td>71.1</td>
<td>58.7</td>
<td>0.0074</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>0.052</td>
<td>0.455</td>
<td>80.5</td>
<td>62.1</td>
<td>0.0069</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>0.059</td>
<td>0.092</td>
<td>39.8</td>
<td>45.9</td>
<td>0.0104</td>
</tr>
<tr>
<td>4.9</td>
<td>13</td>
<td>0.050</td>
<td>0.150</td>
<td>49.0</td>
<td>50.0</td>
<td>0.0092</td>
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<tr>
<td>6</td>
<td>13</td>
<td>0.048</td>
<td>0.242</td>
<td>59.6</td>
<td>53.9</td>
<td>0.0081</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>0.044</td>
<td>0.331</td>
<td>67.2</td>
<td>56.2</td>
<td>0.0075</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
<td>0.042</td>
<td>0.436</td>
<td>74.9</td>
<td>58.4</td>
<td>0.0070</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>0.052</td>
<td>0.071</td>
<td>32.9</td>
<td>40.5</td>
<td>0.0111</td>
</tr>
<tr>
<td>4.9</td>
<td>16</td>
<td>0.048</td>
<td>0.122</td>
<td>44.1</td>
<td>47.3</td>
<td>0.0097</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td>0.041</td>
<td>0.191</td>
<td>53.9</td>
<td>51.6</td>
<td>0.0086</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
<td>0.038</td>
<td>0.270</td>
<td>63.5</td>
<td>55.8</td>
<td>0.0079</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>0.036</td>
<td>0.367</td>
<td>72.3</td>
<td>58.9</td>
<td>0.0073</td>
</tr>
</tbody>
</table>

Table 4.4: **Knitted Spring Tube Parameter Study**

Results are presented in plotted form in Figures 4.29 and 4.30. These data show that by minimizing the courses per inch, or alternatively maximizing the needle radius, the stress is reduced. The lowest stress design of the test matrix was a 16 npt, 4 cpi knitted spring tube with an 0.011 inch diameter wire. This configuration reduces the maximum Von Mises stress by 16.8 percent from the baseline Von Mises stress.
Figure 4.29: Plotted results of parameter study showing cpi vs. force-scaled stress for different number of npt.

Figure 4.30: Plotted results of parameter study showing needle radius vs. force-scaled stress for different number of npt.

4.6 Summary of Knitted Spring Tube Analysis

The single strand knitted spring tubes, made from a single wire knitted in a coarse pattern, showed little interaction between needles. In these cases, wire to wire contact
can be assumed negligible. The baseline spring tube, which has three wires knitted in parallel, shows a transition between free and linked needle behavior. This indicates that at some point during compression of the spring tube, wire to wire contact forces become significant, and should no longer be neglected. Although there is not yet a means of predicting where this transition occurs analytically, the free and linked solutions bound the experimental data. For all three cases considered, the analytically solutions are in close agreement with the experimental observations. The stress solutions of the computational model show high concentrations of stress in the regions near the application of load. These locations are also observed to suffer the most deformation during compression testing. The ability to estimate the stiffness and stresses for new spring tube designs without fabricating prototypes allows for quick screening of geometries and materials. This validated model can be used to design and optimize the knitted spring tube for future, more challenging designs.

References


Appendix A

Finite Element Mesh Algorithm

A.1 Mesh Creator Description

The mesh creator is a set of functions and classes that create a finite element mesh for wire based structures such as knitted spring tubes and canted coil springs. These functions build a solid tubular structure around a path in three dimensional space with 20-node solid brick elements (Figure A.1). Since 20-node brick elements have midside nodes, they may have curved boundaries that are suitable for modeling round surfaces such as the outside of a wire. In order to guarantee that there is continuity throughout the model, elements must be connected completely to each other. Two connected elements must share exactly 8 nodes corresponding to a face.

Figure A.1: A 20-node brick element with nodes labeled a-t.
The 3D path representing the wire structure is transformed into a meshed volume in three steps. First a set of position and direction vectors are created for each point on a path. Second a set of nodes are created on an area normal to the direction vector for each point on the path. Lastly elements are formed by creating ordered lists of nodes. After the mesh creator has defined the geometry as nodes and elements, it links cut faces to meet symmetry conditions and performs additional model preprocessing.

A.2 Mesh Creator Input

The mesh generator requires a set of vectors to describe the location and direction of the wire path at discrete locations. The position vectors are calculated directly from parametric equations defining the geometry of the part to be modeled. If the parametric equations are simple enough that their derivatives with respect to the parameter variable can be calculated easily then the direction vectors can be represented as \( \frac{d\vec{P}}{dt} \) at each point. This vector is normalized if its magnitude is not unity. Otherwise the direction vectors are approximated by a finite difference (Equation A.1).

\[
\vec{v}_t \approx \frac{\vec{P}(t_{i+1}) - \vec{P}(t_i)}{|\vec{P}(t_{i+1}) - \vec{P}(t_i)|}
\] (A.1)

Either the direct of finite difference method is sufficiently accurate to represent the model. A potential problem occurs at a cut face where pattern symmetry is to be applied with coupled nodes. The faces that are coupled must share a normal vector exactly, otherwise erroneous stress concentrations are created. Since the approximation uses a forward difference method to evaluate the direction vector, coupled faces will have slightly different normal vectors if the spring geometry has non-zero second derivatives with respect to the parameter variable. Since this is the case for the spring tube, for which the parametric equations are not easily differentiable, a face that is coupled to another uses the same direction vector that was calculated for the matching face. The canted coil spring parametric equations are easily differentiated.
and have continuous derivatives, so coupled faces are by definition parallel and no correction is required.

### A.3 Creating Nodes

Once the direction vectors tangent to the path are established, three more vectors are created at each point along the path to begin node generation (Figure A.2). The first vector created has a direction that points towards the axis of the spring. For both the spring tube and canted coil spring, the z-axis is the spring axis. This center vector, \( \tilde{v}_c \), is defined by equation A.2:

\[
\tilde{v}_c = \frac{(\hat{P} \cdot \hat{z})\hat{z} - \hat{P}}{|(\hat{P} \cdot \hat{z})\hat{z} - \hat{P}|}
\]

(A.2)

Figure A.2: **Direction vectors along the canted coil spring wire path.**

The tangent vector is the unit direction vector at each point along the path. The normal vector is defined by the cross product of the center vector and the tangent vector \( (\tilde{v}_c \times \tilde{u}_t) \). The normal is rotated about the tangent vector to create points in a plane perpendicular to the tangent vector. The rotation of the normal vector about the tangent vector is accomplished by creating a rotation matrix which operates on the normal vector. The rotation matrix is formed by equation A.3.

\[
R_{ij} = \delta_{ij} \cos (\alpha) - \sin (\alpha) \varepsilon_{ijk} \tilde{u}_k + (1 - \cos (\alpha)) \tilde{u}_i \tilde{u}_j
\]

(A.3)
The points defined by the rotation of the normal vector are arranged in circles and squares of different radii or lengths to create two distinct node patterns. These radii and lengths are chosen such that each element face will have the same area. This criterion helps to establish more uniform characteristic element lengths. The first pattern contains 20 points and is constructed for every odd numbered point along the path (assuming the beginning of the path starts with point 1). The second pattern contains 8 points and is constructed for every even numbered point along the path. The 8 node pattern is for midside nodes only and therefore must always be between two 20 node patterns. This requires that an odd number of points must exist for any path segment.

Figure A.3: **Construction of nodes on a face by vector rotation.**

The nodes created are numbered starting with the node created by the original normal vector at the smallest distance from the center, continuing along each circle or square and then proceeding outward (Figure A.3).
A.4 Creating Elements

With nodes created and numbered, the creation of elements is accomplished by creating ordered lists of nodes. The finite element package used dictates the specific ordering required\(^1\). Nodes from three faces (two odd, one even), make up each element (Figure A.4).

![Odd numbered faces](image1)

![Even numbered faces](image2)

Figure A.4: Construction of elements by sandwiching even and odd faces.

A.5 Linking Cut Faces

Faces are linked to satisfy symmetry conditions, which model an infinite length geometry. Exploiting symmetry greatly reduces model size and complexity and therefore computation time. For surfaces to be linked without incurring erroneous stress concentrations they must share a common normal direction.

Each face is linked to another by coupling their corresponding nodes. Since elements cannot be defined across coupled nodes, the linked face must be odd numbered. Coupling nodes places a restraint that forces their displacements to be identical for each solution step.
A.6 Higher Density Mesh Generation

A higher density mesh was created by a similar method, except that 69 and 25 nodes were created for the odd and even faces respectively (Figure A.5).

Figure A.5: 20 node face (left), 69 node face (right).

To determine whether or not the finite element mesh was sufficiently refined, results were compared between two canted coil spring models with identical geometries but with different mesh densities. A high density knitted spring tube model requires too many nodes to be solved in a reasonable amount of time, so the mesh refinement study was limited to the canted coil spring. A comparison between the force versus displacement results for a medium weight canted coil spring modeled with 300 and 2000 elements shows that the models are nearly identical (Figure A.6). Also, in both the canted coil spring and knitted spring tube models, the element stress solutions have nearly continuous stresses at the element boundaries, thus the errors in the finite element approximation are small. Typical run times for a 300 and 2000 element canted coil spring model are 3 and 100 minutes respectively on a Pentium III machine with 1 GB of RAM. Since the difference in results between the higher and lower density mesh models is negligible, the lower density model was chosen for the canted coil spring and knitted spring tube analyses.
Figure A.6: Comparison of force vs. displacement for medium weight canted coil spring with meshes of 300 and 2000 elements.

References

Bibliography


